ESTIMATING THE RECURRENT NOVA POPULATION IN M31

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1 Introduction and Background

Classical Novae (CNe) are semi-detached binary systems where a late-type secondary star transfers mass to a white dwarf primary (Warner 1995). Outbursts are caused by a thermonuclear runaway in the material accreted onto the surface of the white dwarf (e.g. see Shara 1989, and references therein). Novae exhibit outburst amplitudes of roughly 10 to 20 mag, and can reach peak luminosities as high as $M_V \simeq -9$, making them among the most intrinsically luminous transient sources in the universe (Shafter 2008). Theoretical models show that the properties of the nova outburst are sensitive to both the mass of the white dwarf and its accretion rate, with outbursts arising on massive white dwarfs generally expected to be brighter, and have shorter recurrence times (Livio 1992, also see Townsley & Bildsten 2005).

Over roughly the past century a total of more than 800 optical transient events have been recorded in M31 (Pietsch et al. 2007), mostly discovered as part of large systematic imaging surveys (e.g., Hubble 1929, Arp 1956, Rosino 1973, Ciardullo et al. 1987, Sharov & Askins 1992, Shafter & Irby 2001, Darnley et al. 2004). Almost all of these sources are believed to represent eruptions of classical novae. The impressive dataset of nova positions put together by Pietsch et al. (2007) provides the opportunity to search for multiple outbursts from the same progenitor system, and to characterize the population of recurrent novae (RNe) in M31.

In order to identify RNe *candidates*, we have searched for spatial near coincidences among the 814 novae in the Pietsch et al. (2007) catalog. The study presented here is statistical in nature. Our purpose is *not* to make a case that any particular nova is recurrent, as in an earlier study by Della Valle & Livio (1996), but instead to determine what fraction of the spatial near-coincidences in M31 are likely to represent repeated outbursts of the same nova system (i.e. RNe) and what fraction are expected due to chance. Given that the accuracy of the nova coordinates published over the years is variable (ranging from sub-arcsecond precision for some modern surveys, to several arcsec for some of the older photographic plate data), the number of RNe candidates will depend upon how one defines a spatial coincidence. For our purposes we have defined a variable s, to represent the maximum separation that any two novae may have to be considered a RNe candidate. In their earlier study, Della Valle & Livio (1996) compared nova positions for 16 M31 novae whose positions had been measured by different observers and found a mean error $\sim 0.1'$ (one sigma).

2 The Observed Spatial Distribution of Novae in M31

Figure 1 shows the observed spatial distribution of nova candidates in M31 (814) through the end of 2008. Not surprisingly, the distribution is strongly concentrated toward the center of the galaxy. This is both because the nova distribution generally follows the bulge light (e.g. Shafter & Irby 2001), but also because of selection effects in the nova surveys, which have mostly focused on the inner $\sim$30 arc min of the galaxy. It is clear that the number of chance coincidences is going to be a strong function of spatial position in the galaxy. Given that the surface density of novae in the inner 2 arc min of M31 is $\sim$7 novae per square arcmin (more than 10% of the total number of novae discovered in M31 lie within the inner $2'$!), one can expect a significant number of chance coincidences with plausible values of s as large as $\sim 5''$.

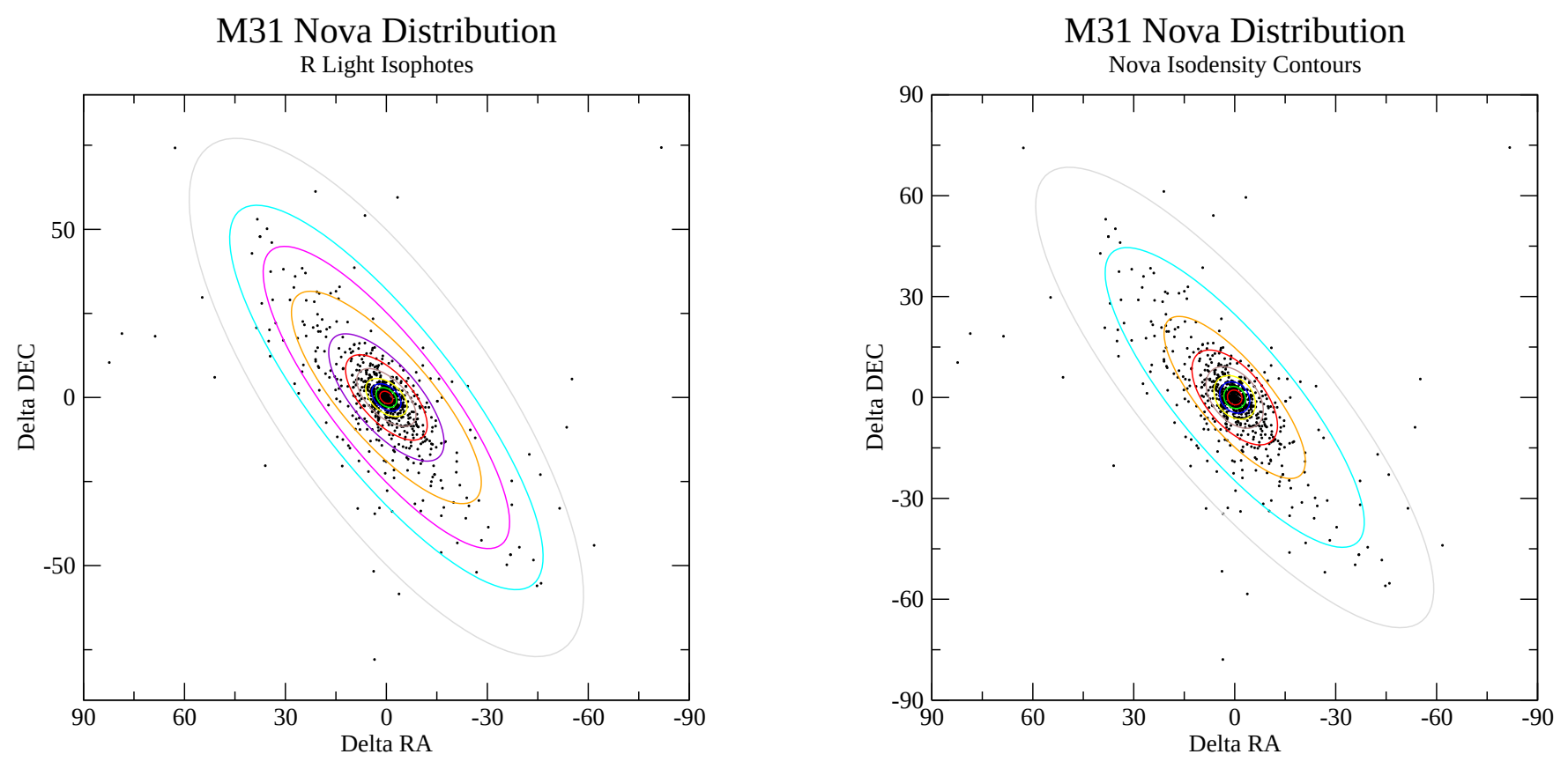

Figure 1: The observed spatial distribution of nova candidates in M31 taken from the summary in Pietsch et al. (2007). The elliptical contours in the left panel represent R band isophotes, while those in the right panel represent nova isodensity contours determined from fits to the observed nova distribution. (the spacing of the contours is not uniform in R light or nova density).

3 M31 Recurrent Nova Candidates

We have cross-correlated the positions of the M31 nova candidates considering a range of positional uncertainties from $s = 2''$ to $s = 6''$. As an example, recorded nova outbursts in M31 within $5''$ of one another are given in Table 1, and their spatial distribution plotted in Figure 2. There are a total of 38 matching pairs. Of these, six pairs are members of three "triples" consisting of three novae all lying within $5''$ of one another. In addition, there is one "quad" consisting of four novae within $5''$ that form six matching pairs. Thus, the total number of matching pairs (38) must be reduced by 11 (2 for each of the three triples, and 5 for the quad) in order to specify the number of RNe candidates, in this case, 27.

Not surprisingly, most candidates of these RNe candidates lie near the center of the galaxy where the nova density is high, and the number of chance coincidences is expected to be large. It is clear that the we must "correct" the 27 RNe candidates for the number RNe candidates that we expect from chance.

For an individual RNe candidate we can estimate the probability of a chance coincidence from the observed separation and an estimate of the nova density in an elliptical annulus of area, A, bracketing the position of the outbursts. The probability of a chance coincidence, P_C, shown in the last column of Table 1, is given by:

$$P_C = 1 - \prod_{i=1}^{n-1} (1 - ix),$$

where n is the number of novae in the annulus and $x = \pi s^2/A$.

In order to make a strong case than any *individual* candidate is in fact an recurrent nova, it would be necessary to examine the original observations to look for similarities in the light curves (if available), and hopefully improve the accuracy of the nova positions. Here, we are interested primarily in a statistical study of the ensemble of RNe candidates. For this, we turn to a discussion of the number of chance coincidences expected for the observed M31 nova spatial distribution.

Table 1: Close ($< 5''$) M31 Nova Pairs

RNe No.	Nova	Recurrence	Separation (arc sec)	Chance Probability
1	M31N1919-09a	M31N1998-06a	1.80	0.025356
2	M31N1924-08a	M31N1987-12a	3.10	0.860896
3	M31N1932-09d	M31N2001-07d	3.63	0.934530
4	M31N1945-09c	M31N1975-11a	0.87	0.000201
5	M31N1953-09b	M31N2004-08a	4.80	0.823353
6	M31N1953-11a	M31N1962-11b	2.90	0.795818
7	M31N1960-12a	M31N1962-11b	4.59	0.982663
8	M31N1961-11a	M31N2005-06c	1.47	0.370164
9	M31N1963-09c	M31N1968-09a	0.54	0.001280
…	…	M31N2001-07b	0.22	0.000219
…	M31N1968-09a	M31N2001-07b	0.67	0.001942
10	M31N1964-12b	M31N1998-07b	3.74	0.918897
11	M31N1966-08a	M31N1968-10c	0.00	0.000000
12	M31N1966-09e	M31N2007-08d	0.36	0.000001
13	M31N1967-11a	M31N2006-02a	4.28	0.976414
14	M31N1967-12a	M31N1993-11c	3.56	0.949175
15	M31N1969-08a	M31N2007-12b	2.13	0.062012
16	M31N1975-09a	M31N1999-01a	3.35	0.565644
17	M31N1977-12a	M31N1998-08a	2.07	0.043887
18	M31N1982-08b	M31N1996-08c	2.99	0.000033
19	M31N1984-07a	M31N2001-10c	1.15	0.202221
…	…	M31N2004-02a	1.63	0.381768
…	…	M31N2004-11f	1.01	0.161114
…	M31N2001-10c	M31N2004-02a	2.78	0.758173
…	…	M31N2004-11f	1.29	0.247869
…	M31N2004-02a	M31N2004-11f	2.09	0.529407
20	M31N1985-10c	M31N1995-12a	2.40	0.600063
…	…	M31N2003-10b	1.60	0.329482
…	M31N1995-12a	M31N2003-10b	3.66	0.889452
21	M31N1986-09a	M31N2006-09b	4.90	0.993322
22	M31N1990-10a	M31N1997-10b	2.26	0.268370
…	…	M31N2007-07a	0.81	0.037690
…	M31N1997-10b	M31N2007-07a	1.82	0.177212
23	M31N1997-10f	M31N2008-08b	0.45	0.038442
24	M31N1997-11k	M31N2001-12b	1.00	0.015331
25	M31N2001-08d	M31N2008-07a	3.44	0.917739
26	M31N2006-11b	M31N2006-12d	0.35	0.022199
27	M31N2006-12c	M31N2007-07e	4.02	0.964194

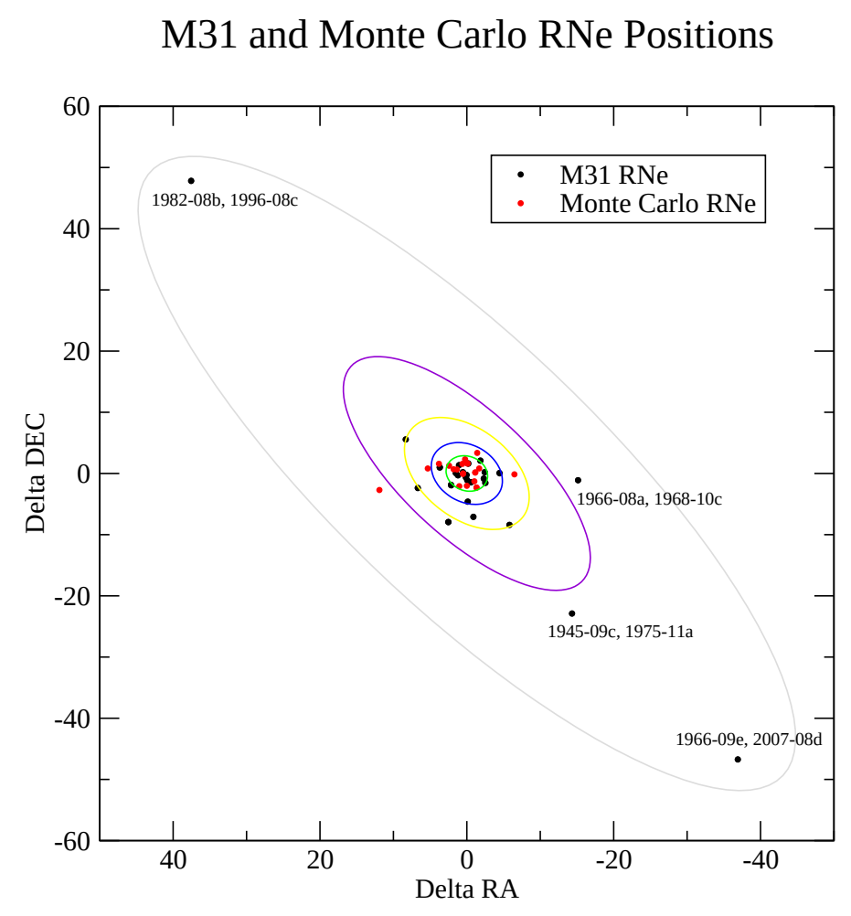

Figure 2: The spatial distributions of the M31 RNe candidates (black dots) are compared with that of a single Monte Carlo simulation run (red dots). It is clear that the majority of RNe Candidates within $\sim 5''$ of the nucleus are likley due to chance coincidences.

4 The Monte Carlo Simulations

We begin by estimating the number of chance coincidences in the M31 nova distribution through a series of numerical "Monte Carlo" simulations. In order to take into account the effect of spatial position in the galaxy, we have divided the galaxy into a series of annular regions. If the actual nova distribution followed the background light and if the surveys used to discover the novae were spatially complete, then a suitable set of M31 isophotes would be ideal for defining our annuli. However, given these caveats, we decided a better approach would be to use the observed M31 nova distribution to define a series of nova isodensity contours. A drawback of this approach is that it is difficult to define these contours in the outer region of the galaxy where the observed density is low. To improve the statistics we have assumed that the M31 nova distribution is symmetric with respect to reflection about both the major and minor axis of the galaxy and folded the data about these axes. Representative nova isodensity contours resulting from fits to the folded data are shown in the right panel of Figure 1 superimposed on the observed nova distribution.

Within a series of annuli of varying ellipticity defined by the nova isodensity contours, we randomly distribute a number of simulated novae matching the number of novae observed to erupt within the confines of that annulus. After summing the contributions from each annulus, we then searched for matching pairs (simulated RNe) just like we did for the observed distribution. We repeated this process 10^6 times for each of five separations between $s = 2''$ and $s = 6''$. The resulting distribution of matches for $s = 5''$ is shown in Figure 3.

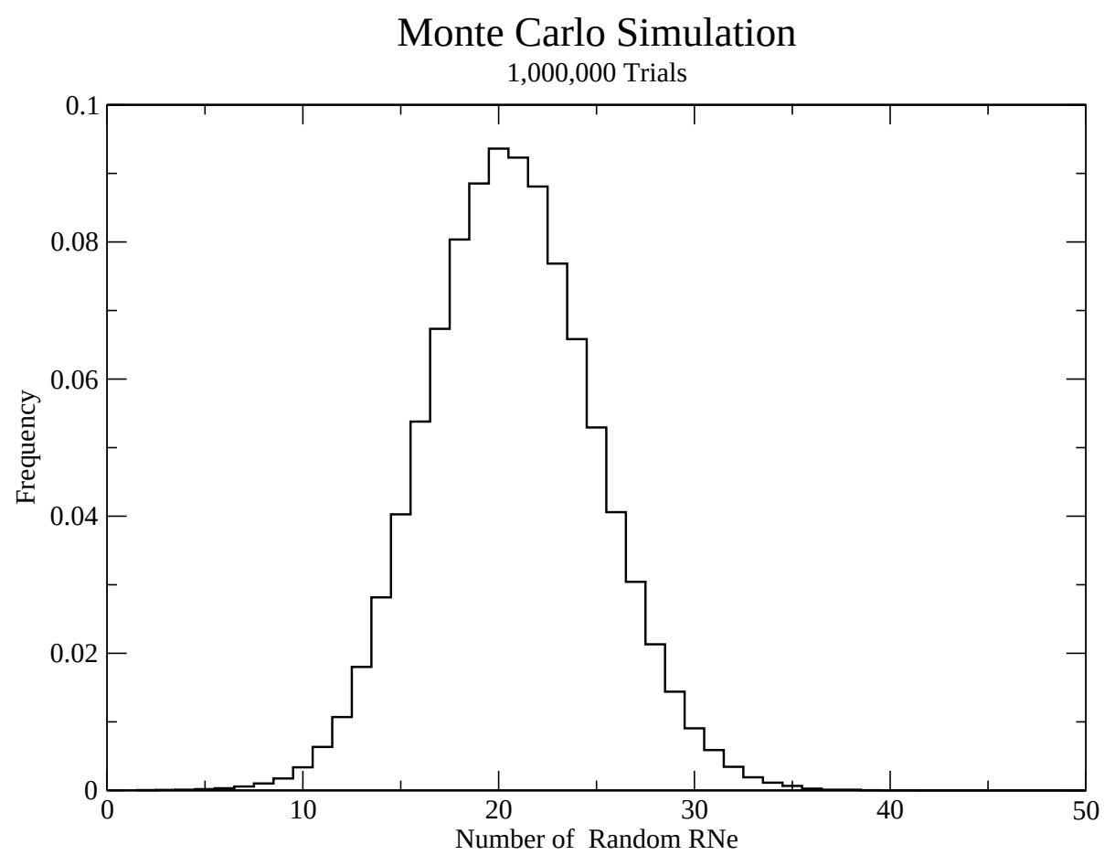

Figure 3: The Monte Carlo Distribution showing the number of RNe that are expected by chance. The most likely number of RNe expected by chance is 20.

5 Analytical Calculation of Chance Coincidences

As a check on the Monte Carlo Results, we have also calculated the number of chance coincidences expected for a given value of the separation, s. We have included both the possibility of the formation of pairs and of triples (3 novae all lying within s of one another), but have neglected higher order multiples. If we let n be the number of novae expected in a given annulus of area, A, and as before $x = \pi s^2/A$, we find the total number of pairs, P, is given by

$$P = \frac{x}{1-\alpha}\left[(n - \frac{1-\alpha^n}{1-\alpha}\right],$$

where $\alpha = 1 - 1.7595x$. In a similar fashion, the expected number of "triple" matches, T, is found to be:

$$T = \frac{1}{3}\left[\frac{n(n-1)x}{2} - P\right].$$

The total number of pairs, $P_T = P + 3T$, is given by:

$$P_T = \sum_{n=1}^{n}(n-1)x = \frac{n(n-1)x}{2}.$$

The number of RNe expected from our analytical calculations, N_A, is simply the sum of P and T. Noting that $P = P_T - 3T$, we have more simply:

$$N_A = \frac{2}{3}P + \frac{1}{3}\left[\frac{n(n-1)x}{2}\right].$$

After inserting the values of n and x appropriate for each annulus, we find that $N_A = 20.81$, which is in excellent agreement with the mean of the Monte Carlo distribution $N_{MC} = 20.24$.

6 Discussion and Conclusions

The results of the Monte Carlo and analytical calculations for $s = 5''$ suggest that of the 27 RNe candidates from the observed data, $\sim$20 are expected to occur by chance coincidence, with almost all of these occurring within a few arcmin of the nucleus. Thus, it appears that, statistically, one would expect that only $\sim$7 of the observed close pairs (within $5''$) actually represent recurrent nova systems.

To explore the effect of the adopted error separation, s, on the predicted number of RNe systems, we have carried out our analysis for additional separations of $s = 2'', 3'', 4''$, and $6''$. The results of this exercise are shown in Figure 4. The predicted number of RNe systems decreases with increasing error separation, s, as the number of chance coincidences increases relative to the number of RNe. For small separations, $s \lesssim 3''$, the number of RNe appears to level off at a value of $\sim$9 RNe, which we adopt as a rough estimate of the number of RNe systems *observed* to erupt in M31. A determination of the ratio of recurrent nova outbursts to classical nova outbursts will require that the observed numbers be corrected for the discovery efficiencies of both RNe and CNe in the M31 nova surveys.

Finally, we wish to stress again that our analysis is purely statistical in nature, and is meant solely to correct the number of M31 RNe candidates determined from positional near coincidence for the number expected by chance. A proper assessment of whether or not a given close pair is in fact a recurrent nova will require a critical examination of all available data, including astrometry, light curve morphology, and spectroscopic class (Williams 1992), for the pair of novae.

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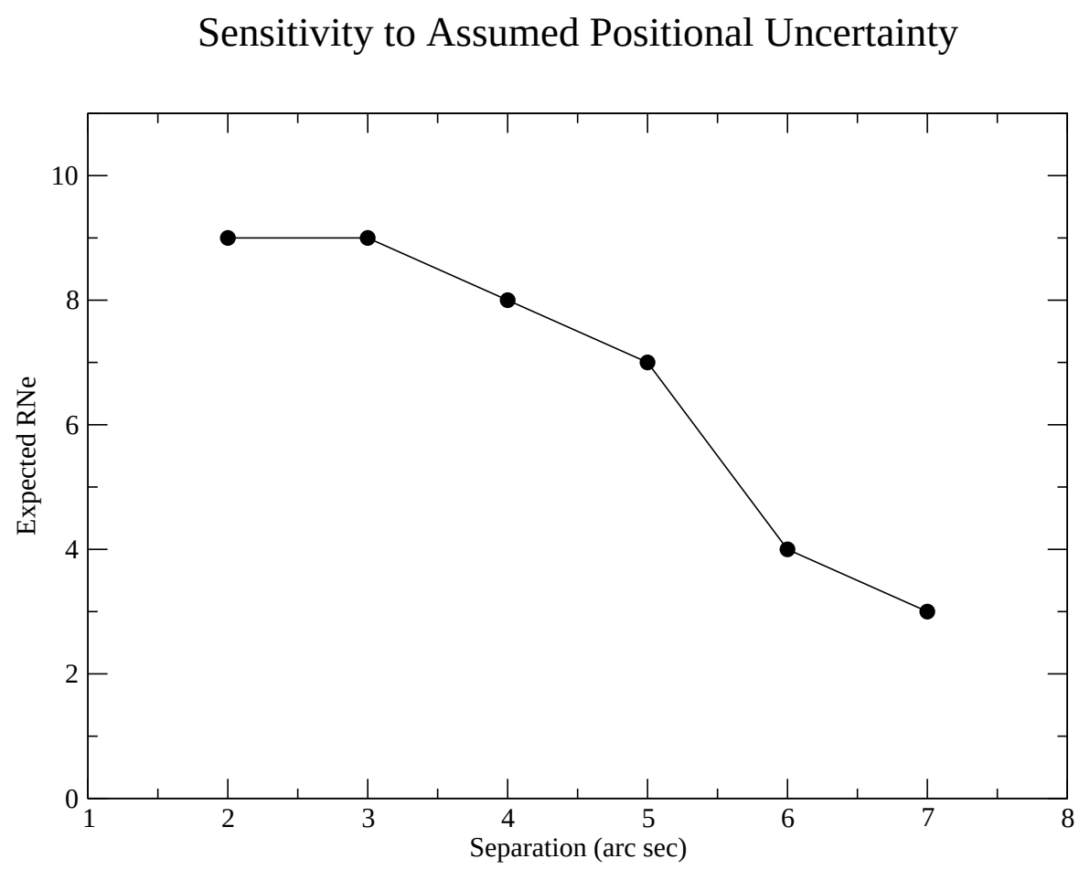

Figure 4: The variation of the expected number of RNe as a function of the separation, s, used to define the positional uncertainty.

7 References

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